NEW STEERING STRATEGIES FOR THE USNO MASTER CLOCKS

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Abstract

All U.S. Naval Observatory (USNO) steering situations involve compromises to minimize the degradation of short-term stability of a steered clock while gaining maximal benefits from the long-term stability of the reference. In the case of steering UTC(USNO) to UTC, extra complications arise due to the 30-day data interval and the 15-day delay associated with the transfer of new information. A technique that minimizes the amount of control required to steer the USNO mean to UTC will be presented. Different strategies designed for optimal steering of UTC(USNO) and a backup master clock system located at the USNO will be described. Some of these strategies involve steering a maser to an intermediate mean that is steered to an extrapolation of UTC. Examples of optimal steering on real data will be reported.

INTRODUCTION

The United States Naval Observatory (USNO) is the time reference for the Department of Defense (DOD). This requires the USNO Master Clock systems to be accurate, stable, and robust. Accuracy is accomplished by synchronizing and syntonizing the USNO Master Clock with respect to the international time scale UTC(BIPM). Frequency stability is insured by limiting control efforts on the USNO Master Clock. Robustness is attained by setting up autonomous remote systems, such as the Alternate Master Clock (AMC) in Colorado, that are aligned to the USNO Master Clock. In the first section we cover the theory behind a minimal control technique, along with data simulations and planned applications. The following section covers a design that utilizes an intermediate time scale, or mean, in a control situation that maintains the tracking of a remote system to the USNO Master Clock.

MINIMIZING CONTROL EFFORT

This control design technique is intended to gently steer the USNO Mean to the international time scale defined by the BIPM. A block diagram describing the steps involved in steering to the BIPM is given in Figure 1. Data from the BIPM are published monthly with a time lag of approximately 15 days. This non-causal system requires the prediction of the present time and frequency offsets utilizing the 15-day-old data. This prediction is accomplished by extrapolating a linear fit done to the last 50 days of published time difference data [4]. After the present time and frequency offsets are predicted from the given data, the sequence of frequency steers minimizing the control effort can be determined.

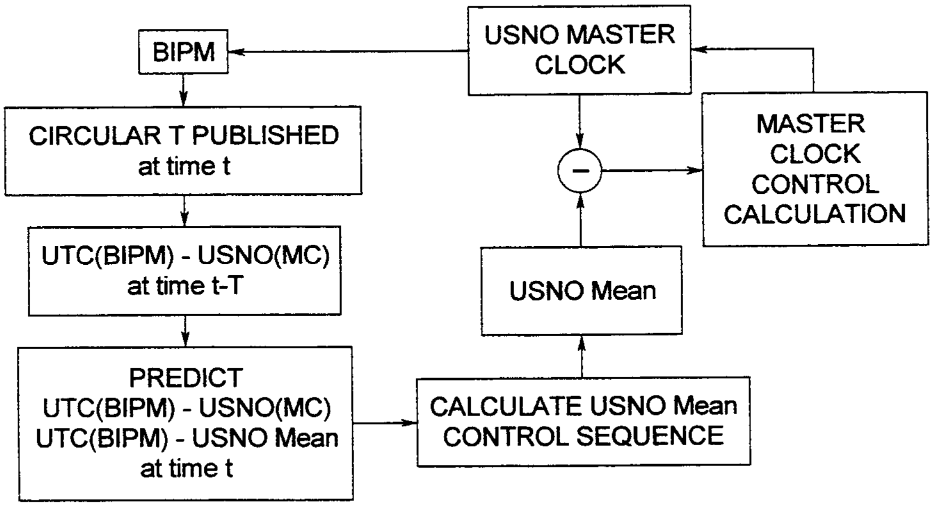

Figure 1. USNO Steering to BIPM

The linear noise free state space representation has the general form ([1],[5])

$$\mathbf{X}(k+1) = \mathbf{\Phi X}(k) + \mathbf{BU}(k). \tag{1}$$

The state space representation for an ideal, noise-free, frequency standard controlled by discrete frequency steers $u(k)$, is given by

$$\begin{bmatrix} x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} \tau \\ 1 \end{bmatrix} u(k), \tag{2}$$

where x and y correspond , respectively, to the time and fractional frequency difference between a steered frequency standard and a reference, and τ corresponds to the time interval between measurement updates.

We wish to minimize the control effort, or so-called control energy,

$$\frac{1}{2}\sum_{k=0}^{N-1} u^2(k) \tag{3}$$

necessary to drive the state values, x and y, to zero in N steps.

As shown in [5], after N sampling periods the state $\mathbf{X}$ can be represented by

$$\mathbf{X}(N) = \mathbf{\Phi}^N \mathbf{X}(0) + \mathbf{\Phi}^{N-1}\mathbf{B}u(0) + \mathbf{\Phi}^{N-2}\mathbf{B}u(1) + \cdots + \mathbf{\Phi B}u(N\text{-}2) + \mathbf{B}u(N\text{-}1). \tag{4}$$

Setting $\mathbf{X}(N) = 0$ in the preceding equation and solving for $\mathbf{X}(0)$ gives

$\mathbf{X}(0) = -\mathbf{FU}$ where

$$\mathbf{F} = \left[\Phi^{-1}\mathbf{B} \vdots \Phi^{-2}\mathbf{B} \vdots \cdots \vdots \Phi^{-N}\mathbf{B}\right] \quad \text{and} \quad \mathbf{U} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}. \tag{5}$$

As proven in [5] the minimal control effort solution for **U** is found by applying the right pseudo-inverse to solve for **U** in (5),

$$\mathbf{U} = \mathbf{F}^{\dagger}(\mathbf{F}\mathbf{F}^{\dagger})^{-1}\,\mathbf{X}(0)\,, \tag{6}$$

where † denotes the conjugate transpose.

We will now apply this to the frequency standard model. From (2) we have

$$\Phi^{-N}\mathbf{B} = \begin{bmatrix} -(N-1)\tau \\ 1 \end{bmatrix} \tag{7}$$

then

$$\mathbf{F} = \begin{bmatrix} 0 & -\tau & -2\tau & \cdots & -(N-1)\tau \\ 1 & 1 & 1 & & 1 \end{bmatrix}. \tag{8}$$

The solution for **U** with respect to minimizing the control effort given by (6) is

$$\mathbf{U} = -\frac{6}{N(N+1)} \begin{bmatrix} \frac{1}{\tau} & \frac{1}{3}(2N-1) \\ \frac{1}{\tau}(1-2\frac{1}{N-1}) & -1+\frac{1}{3}(2N-1) \\ \frac{1}{\tau}(1-2\frac{2}{N-1}) & -2+\frac{1}{3}(2N-1) \\ \vdots & \vdots \\ -\frac{1}{\tau} & -(N-1)+\frac{1}{3}(2N-1) \end{bmatrix} \begin{bmatrix} x(0) \\ y(0) \end{bmatrix}. \tag{9}$$

The matrix elements of **U** are given by

$$u_{ij} = -\frac{6}{N(N+1)}\left\{\frac{1}{\tau}\left(1-\frac{2(i-1)}{N-1}\right)x(0)\delta_{1j} + \left[(1-i)+\frac{1}{3}(2N-1)\right]y(0)\delta_{2j}\right\} \tag{10}$$

for i = 1 ... N and δ_{ij}'s are Kronecker deltas.

The individual steers for this application can also be expressed in the linear form,

$$u(k) = -\frac{6x(0) + 4(N-1)y(0)\tau}{N(N-1)\tau} + \frac{6[2x(0) + y(0)(N-1)\tau]}{N(N^2-1)\tau}k \tag{11}$$

for $k = 1 \ldots N$.

Plots of simulations using several different control update time intervals τ in order to bring an ideal frequency standard with offsets of 5 ns in time and $3*10^{-15}$ in fractional frequency in one month (corresponding to the update period for BIPM data) are shown in Figure 2. An advantage to the predetermined nature of this control technique is that the amount of time and/or frequency offsets to be removed can be reduced if analysis shows the control perturbations are unacceptably large. In order to gain insight on the control perturbations, the Allan deviation of the frequency steering sequence is calculated assuming the control is implemented on an ideal noiseless frequency standard. The Allan deviation given in Figure 3 of the hourly steered simulation data in Figure 2 remains at or below $1*10^{-15}$.

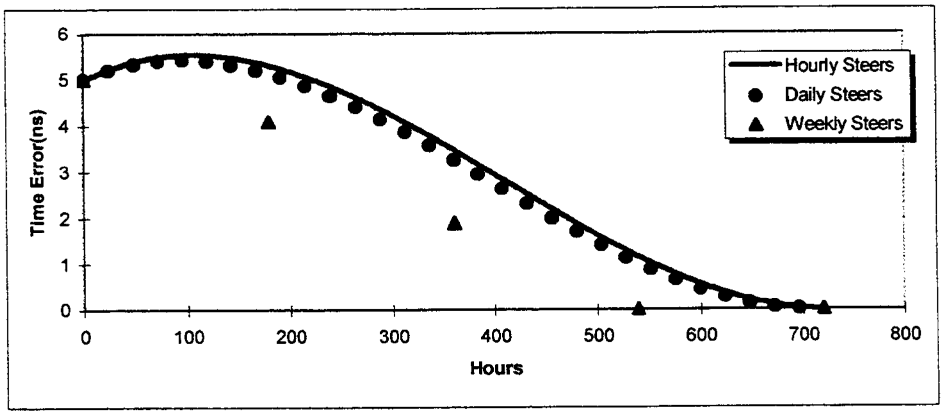

Figure 2. Minimal Control Effort Simulations: Initial Time Error=5 ns, Freq. Error=3E-15

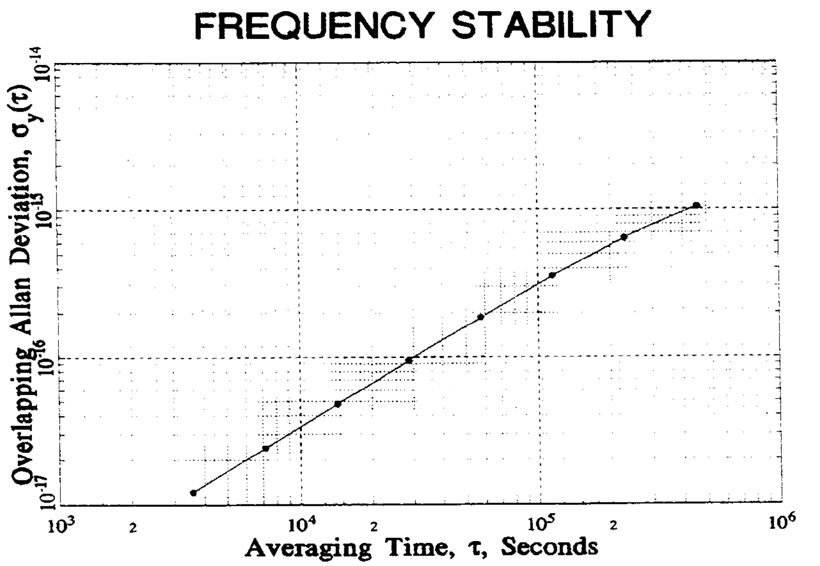

Figure 3. Allan Deviation of Hourly Control Perturbations in Figure 2.

INTERMEDIATE MEAN

The intermediate mean concept was formed in an attempt to create autonomous and robust remote timing systems that are capable of becoming Master Clock reference signals should the need arise. Figure 4 shows a block diagram of the concept. The intermediate mean is a paper clock time scale that is calculated using only the frequency standards located at the remote site. Robustness is gained primarily by having the intermediate mean act as a buffer. The buffer is formed by controlling a remote frequency standard to the remote calculated mean and then steering that mean to the reference signal (Master Clock). It is difficult operationally to determine when the reference signal has gone bad, unless the signal is lost altogether. The remote steered frequency standard is buffered by the response of the mean to any anomalous events of the Master Clock. This method also has the advantage that it is very straightforward for the steering of the remote reference to the mean to continue if the connection to the Master Clock is lost.

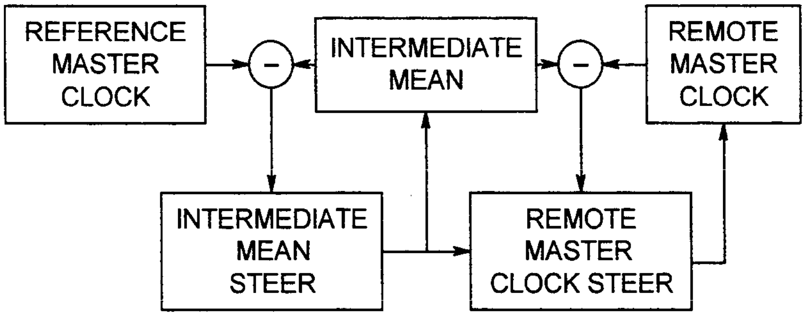

Figure 4. Intermediate Mean

The noise-free space representation for this system is

$$\begin{bmatrix} x_r(k+1) \\ y_r(k+1) \\ x_m(k+1) \\ y_m(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(k) \\ y_r(k) \\ x_m(k) \\ y_m(k) \end{bmatrix} + \begin{bmatrix} -\tau & \tau \\ -1 & 1 \\ 0 & -\tau \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_r(k) \\ u_m(k) \end{bmatrix}, \tag{12}$$

where the subscript r corresponds to the difference between the intermediate mean and the remote steered clock and the subscript m corresponds to the difference between the Master Clock and the intermediate mean.

The initial implementation created an autonomous backup system to the USNO Master Clock in Washington DC in a separate building on the USNO grounds containing several hydrogen masers and cesium standards. A mean was calculated from the data gathered from these standards and that mean was steered to the Master Clock. A hydrogen maser in the remote building was chosen to be the source for the remote Master Auxiliary Output Generator (AOG) frequency synthesizer that was steered to the calculated time-scale mean. Two independent Kalman filters calculate the state estimates for the two pairs of time and frequency difference data.

The matrix control gain,

$$\mathbf{G} = -\begin{bmatrix} 9.75*10^{-9} & 8.37*10^{-3} & 2.03*10^{-9} & 7.52*10^{-3} \\ -2.00*10^{-9} & -8.59*10^{-4} & 9.63*10^{-9} & 3.15*10^{-2} \end{bmatrix}, \qquad (13)$$

was calculated using the linear quadratic technique, as outlined in [2] and [3], with cost functions

$$\mathbf{W}_R = 10^{12}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and}$$

$$\mathbf{W}_Q = \begin{bmatrix} 10^{-4} & 0 & 0 & 0 \\ 0 & 10^{6} & 0 & 0 \\ 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 10^{9} \end{bmatrix}.$$

The control vector is defined by the linear matrix equation $\mathbf{U} = -\mathbf{G}\hat{\mathbf{X}}$, where $\hat{\mathbf{X}}$ is the state estimate.

Plots of the data between the USNO Master Clock and the intermediate mean, and between the intermediate mean and the AOG are shown in Figure 5. The system robustness is evident from the response to manual shifts in the intermediate mean, and is enhanced by the high weight given to frequency stability in the cost function used to compute the mean. Figure 6 gives the Allan deviation between the USNO Master Clock and the remote AOG. The performance is approximately what we expect between two hydrogen masers showing that the design has kept the system tracking without unduly degrading the frequency stability of the steered standard.

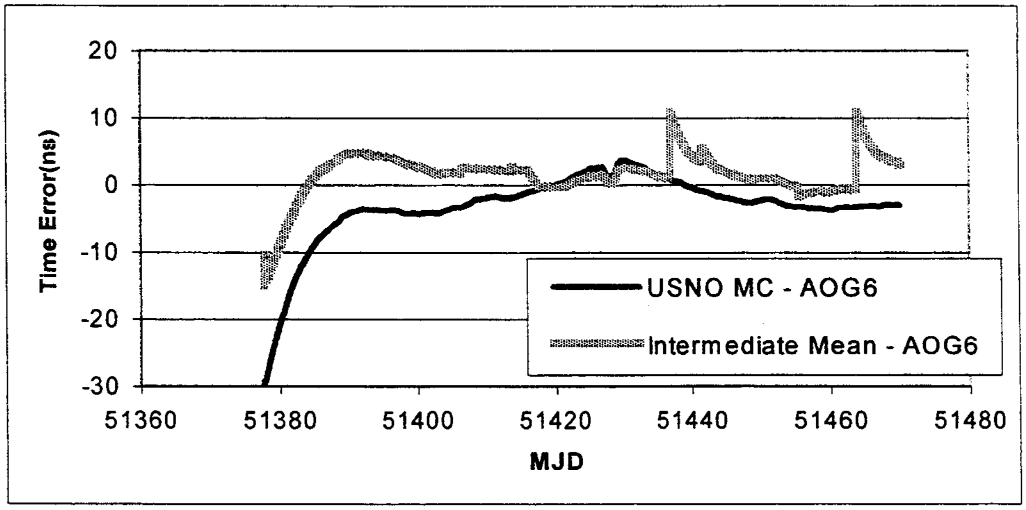

Figure 5. Intermediate Mean Results

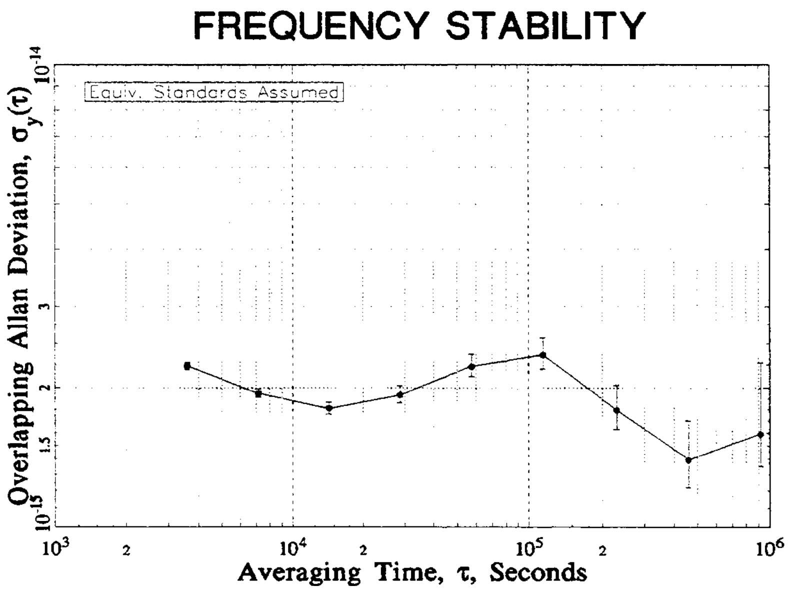

Figure 6. Allan Deviation of USNO MC – AOG6 (after initial settling)

CONCLUSION

The control effort design described creates a predetermined control sequence that minimizes the amount of steering necessary to bring the time and frequency offsets to zero in a given amount of time and number of steps. This design is particularly useful in the application of steering frequency standards and/or time scales to the BIPM. Initial data presented on an intermediate mean control systemwere shown to be an effective design strategy in the application of robust remote backup timing systems.

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Questions and Answers

CAPTAIN MICHAEL RIVERS (USAF 2SOPS): My question is for Paul Koppang. Do you have any statistical evidence of how your new steering algorithm differs from the old steering algorithm in terms of time stability versus frequency stability? Because I think that's what 2SOPS and a lot of the users we deal with are more concerned with.

PAUL KOPPANG (Datum): Not yet. The analysis is starting, and maybe Demetrios might want to say something about that. Basically, it's a theoretical analysis right now and it hasn't been put into operation yet.

DEMETRIOS MATSAKIS (USNO): Actually, we have done some simulations. The amount of control, which is the amount of time that we are steering a maser falls by a factor of two or three. So that would translate directly into a frequency stability improvement right there.

There's different regimes of time stability. If you keep the frequency stable on the short term, it will integrate up and correspond to time stability. On a longer term, our time stability is limited by our ability to predict the difference between our clocks and the BIPM's. And that is unaffected. So in the long term, by which I mean months, this algorithm would have no effect at all. On the short term, it leads to an improvement in all levels, and that's backed up by simulations.

STEVEN HUTSELL (USNO AMC): Paul, as you remember when you worked for the Observatory, we were toying around on the idea of the intermediate mean at the AMC. You probably are already current on this, but that's actually been operational for 14 months; there have been some modifications. And obviously, there's always going to be the debate between the various customers and users about which are the priorities in terms of optimization. And obviously, one of my biases is going to be towards stability, obviously, versus accuracy in an absolute sense. But I recognize it's something that needs to be balanced.

In any case, we do have data regardless of whether everyone agrees on whether it's optimally prioritized for the goals. We do have data and once you get back in town, if you want to give me a phone call, I'll be glad to show you some information. It's not going to be optimal for everyone, but we are not seeing the steps and jumps that I believe I saw in some of your plots. And I may be able to provide some information on lessons learned that may help out. They may not, but give me a call and I'll see what I can do.